CONTINUITY & DIFFERENTIABILITY-MODULE 5

RECAP

PROPERTIES OF LOGARITHM

Logarithm Properties

- $\log_a xy = \log_a x + \log_a y$
- $\log_a \frac{x}{y} = \log_a x - \log_a y$
- $\log_a x^n = n \log_a x$
- $\log_a b = \frac{\log_c b}{\log_c a}$
- $\log_a b = \frac{1}{\log_b a}$

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a a^r = r$

log 1 = 0
log e = 1
log 10 = 1

APPLY LOG.ONBOTH SIDES

- $(sinx)^y = (siny)^x$
- Y = 2^{sinx}
- Y = $x^{\log x}$
- Y = $\sqrt{(x-5)(x+8)}$
- xy = e^{x-y}
- $y = (x-2)^3(2x-9)^4$

Use
log. rules
and answer
quickly

Where we will apply log.differentiation ?

- 1)Y = $[f(x)]^{g(x)}$ Eg: $(sinx)^{x}$
- 2) Funtions involving many operations like multiplication ,division,square root……

Eg: $\sqrt{\frac{(x-2)(x+3)^4}{(2x-5)^3}}$

Q.1

$$y = x^{2x}$$

$$\log y = \log\left(x^{2x}\right)$$

$$\log y = 2x \log x$$

$$\frac{d}{dx}[\log y] = \frac{d}{dx}[2x \log x]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \log x \cdot \frac{d}{dx}[2x] + 2x \cdot \frac{d}{dx}[\log x]$$

$$= \log x \cdot 2 + 2x \cdot \frac{1}{x}$$

$$= 2 \log x + 2$$

$$\frac{dy}{dx} = y\,(2 \log x + 2)$$

$$= x^{2x}\,(2 \log x + 2)$$

$$= 2x^{2x} \log x + 2x^{2x}$$

Q.2

$\therefore y = (x + 1)^3 (x + 2)^4 (x + 3)^5$

Solution:

RULE: log(ABC) = log A + log B + log C

We know that

$$y = (x + 1)^3 (x + 2)^4 (x + 3)^5$$

By taking log on both sides

$$\log y = \log((x + 1)^3 (x + 2)^4 (x + 3)^5)$$

It can be written as

$$\log y = \log[(x + 1)^3] + \log[(x + 2)^4] + \log[(x + 3)^5]$$

$$\log y = 3\log(x + 1) + 4\log(x + 2) + 5\log(x + 3)$$

By differentiating both sides w.r.t.x

$$\frac{1}{y}\frac{dy}{dx} = 3.\frac{1}{x+1}.1 + 4.\frac{1}{x+2}.1 + 5.\frac{1}{x+3}.1$$

On further calculation

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{x+1} + \frac{4}{x+2} + \frac{5}{x+3}$$

We can write it as

$$\frac{dy}{dx} = y\left(\frac{3}{x+1} + \frac{4}{x+2} + \frac{5}{x+3}\right)$$

Substituting the value of y

$$\frac{dy}{dx} = (x + 1)^3 (x + 2)^4 (x + 3)^5 \left[\frac{3}{x+1} + \frac{4}{x+2} + \frac{5}{x+3}\right]$$

Q.3

$$\therefore \mathbf{y} = \frac{(\mathbf{x}+\mathbf{1})^2\sqrt{\mathbf{x}-\mathbf{1}}}{(\mathbf{x}+\mathbf{4})^3.\mathbf{e}^{\mathbf{x}}}$$

Solution:

We know that

$$y = \frac{(x+1)^2\sqrt{x-1}}{(x+4)^3.e^x}$$

By taking log on both sides

$$logy = log[\frac{(x+1)^2(x-1)^{\frac{1}{2}}}{(x+4)^3.e^x}]$$

It can be written as

$$logy = log((x+1)^2(x-1)^{\frac{1}{2}}) - log((x+4)^3.e^x)$$

So we get

$$logy = log((x+1)^2) + log[(x-1)^{\frac{1}{2}}] - log[(x+4)^3] - loge^x$$

$$logy = 2log(x+1) + \frac{1}{2}log(x-1) - 3log(x+4) - xloge$$

$$logy = 2log(x+1) + \frac{1}{2}log(x-1) - 3log(x+4) - x$$

By differentiating both sides w.r.t.x

$$\frac{1}{y}\frac{dy}{dx} = 2.\frac{1}{x+1} + \frac{1}{2}.\frac{1}{x-1} - 3.\frac{1}{x+4} - 1$$

It can be written as

$$\frac{dy}{dx} = y[\frac{2}{x+1} + \frac{1}{2(x-1)} - \frac{3}{x+4} - 1]$$

By substituting the value of y

$$\frac{dy}{dx} = \frac{(x+1)^2\sqrt{x-1}}{(x+4)^3.e^x}[\frac{2}{x+1} + \frac{1}{2(x-1)} - \frac{3}{x+4} - 1]$$

RULE: $\log(\sqrt{\frac{AB}{C}}) = \frac{1}{2}(\log A + \log B - \log C)$

Q.4

$y = sin2x\ sin3x\ sin4x$

By taking log on both sides

$log y = log[sin2x\ sin3x\ sin4x]$

It can be written as

$log y = log(sin2x) + log(sin3x) + log(sin4x)$

By differentiating both sides w.r.t.x

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{sin2x}.cos2x.2 + \frac{1}{sin3x}.cos3x.3 + \frac{1}{sin4x}.cos4x.4$$

$$\frac{1}{y}\frac{dy}{dx} = 2cot2x + 3cot3x + 4cot4x$$

It can be written as

$$\frac{dy}{dx} = y[2cot2x + 3cot3x + 4cot4x]$$

By substituting the value of y

$$\frac{dy}{dx} = sin2x\ sin3x\ sin4x[2cot2x + 3cot3x + 4cot4x]$$

Y = UVW

$$\frac{dy}{dx} = UVW' + UWV' + VWU'$$

$$y = sin2x\ sin3x\ sin4x$$

$$\frac{dy}{dx} = \text{sin2x.sin3x}(sin4x)' + \text{sin2x.sin4x}(sin3x)' + \text{sin3x.sin4x}(sin2x)'$$

= ....

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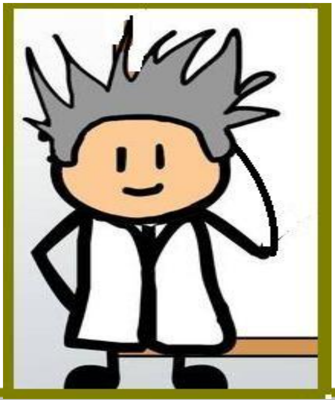

Q.5

$$\therefore y = \frac{x\cos^{-1}x}{\sqrt{1-x^2}}$$

Solution:

We know that

$$y = \frac{x\cos^{-1}x}{\sqrt{1-x^2}}$$

By taking log on both sides

$$\log y = \log\left[\frac{x\cos^{-1}x}{\sqrt{1-x^2}}\right]$$

It can be written as

$$\log y = \log(x\cos^{-1}x) - [\log(1-x^2)^{\frac{1}{2}}]$$

$$\log y = \log x + \log(\cos^{-1}x) - \frac{1}{2}\log(1-x^2)$$

By differentiating both sides w.r.t.x

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{\cos^{-1}x}\left[-\frac{1}{\sqrt{1-x^2}}\right] - \frac{1}{2}\frac{1}{1-x^2}(-2x)$$

It can be written as

$$\frac{dy}{dx} = y\left[\frac{1}{x} - \frac{1}{\sqrt{1-x^2}\cos^{-1}x} + \frac{x}{1-x^2}\right]$$

By substituting the value of y

$$\frac{dy}{dx} = \frac{x\cos^{-1}x}{\sqrt{1-x^2}}\left[\frac{1}{x} - \frac{1}{\sqrt{1-x^2}\cos^{-1}x} + \frac{x}{1-x^2}\right]$$

Q.6

$y = x^x - 2^{sinx}$

$Take\ x^x = u\ and\ 2^{sinx} = v$

$So\ we\ get\ y = u - v$

By differentiating both sides w.r.t.x

$$\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$$

We get

$u = x^x$

By taking log on both sides

$logu = log(x^x)$

Here

$logu = xlogx$

By differentiating both sides w.r.t.x

$$\frac{1}{u}.\frac{du}{dx} = x.\frac{1}{x} + logx.1$$

It can be written as

$$\frac{du}{dx} = u(1 + logx)$$

Substituting the value of u

$$\frac{du}{dx} = x^x(1 + logx)$$

$v = 2^{\sin x}$

Taking logarithm on both the sides with respect to x, we obtain

$\log v = \sin x \cdot \log 2$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\cdot\frac{dv}{dx} = \log 2 \cdot \frac{d}{dx}(\sin x)$$

$$\Rightarrow \frac{dv}{dx} = v \log 2 \cos x$$

$$\Rightarrow \frac{dv}{dx} = 2^{\sin x} \cos x \log 2$$

$$\therefore \frac{dy}{dx} = x^x(1 + \log x) - 2^{\sin x} \cos x \log 2$$

Don't take log on both sides, because log(m+n) ≠log m +log n

Q.7

log (m+n) ≠log m + log n

$y = (\log x)^x + x^{\log x}$

Solution:

We know that

$$y = (logx)^x - x^{logx}$$

Take $(logx)^x = u$ *and* $x^{logx} = v$

So we get $y = u + v$

By differentiating both sides w.r.t.x

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

We get

$$u = (logx)^x$$

By taking log on both sides

$$logu = log([log]^x)$$

Here

$$logu = xlog(logx)$$

By differentiating both sides w.r.t.x

$$\frac{1}{u}.\frac{du}{dx} = x.\frac{1}{logx}.\frac{1}{x} + log(logx).1$$

It can be written as

$$\frac{du}{dx} = u(\frac{1}{logx} + log(logx))$$

Substituting the value of u

$$\frac{du}{dx} = (logx)^x(\frac{1}{logx} + log(logx))$$

We know that

$$v = x^{logx}$$

By taking log on both sides

$$logv = log(x^{logx})$$

$$logv = logx.logx$$

By differentiating both sides w.r.t.x

$$\frac{1}{v}.\frac{dv}{dx} = logx\frac{1}{x} + logx\frac{1}{x}$$

So we get

$$\frac{dv}{dx} = v.\frac{2}{x}logx = x^{logx}.\frac{2logx}{x}$$

By substituting the value of $\frac{du}{dx}$ *and* $\frac{dv}{dx}$

$$\frac{dy}{dx} = (logx)^x[\frac{1}{logx} + log(logx)] + x^{logx}.\frac{2logx}{x}$$

Q.8

33. y = x ^{sinx} + (sin x) ^{cosx}

Solution:

$We\ know\ that$

$y = x^{sinx} - (sinx)^{cosx}$

$Take\ (x)^{sinx} = u\ and\ (sinx)^{cosx} = v$

$So\ we\ get\ y = u + v$

$By\ differentiating\ both\ sides\ w.r.t.x$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$We\ get$

$u = x^{sinx}$

$By\ taking\ log\ on\ both\ sides$

$logu = sinxlogx$

$By\ differentiating\ both\ sides\ w.r.t.x$

$So\ we\ get$

$$\frac{1}{u}.\frac{du}{dx} = sinx.\frac{1}{x} + logx.cosx$$

$It\ can\ be\ written\ as$

$$\frac{du}{dx} = u(\frac{sinx}{x} + logx.cosx)$$

$Substituting\ the\ value\ of\ u$

$$\frac{du}{dx} = x^{sinx}(\frac{sinx}{x} + logx.cosx)$$

$v = (sinx)^{cosx}$

$By\ taking\ log\ on\ both\ sides$

$logv = cosx.log(sinx)$

$By\ differentiating\ both\ sides\ w.r.t.x$

$$\frac{1}{v}.\frac{dv}{dx} = cosx\frac{1}{sinx}.cosx + log(sinx).(-sinx)$$

$$\frac{dv}{dx} = v(cosxcotx - log(sinx).sinx) = (sinx)^{cosx}(cosxcotx - sinxlog(sinx))$$

$$\frac{dy}{dx} = x^{sinx}(\frac{sinx}{x} + logx.cosx) + (sinx)^{cosx}(cosxcotx - sinxlog(sinx))$$

Log (m+n) ≠ log m + log n

- HOME WORK
- EX5.5

  2, 8,11,12,14

THANK YOU…….will continue tomorrow